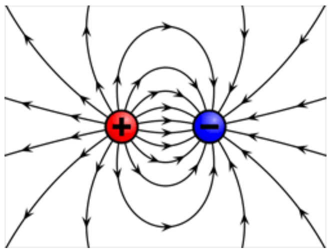
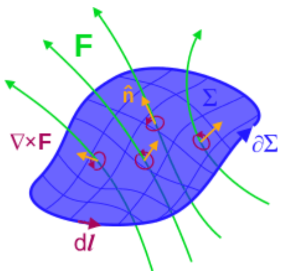
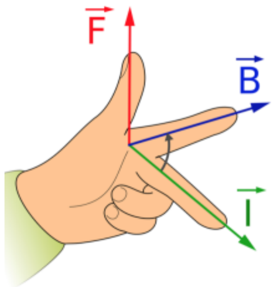
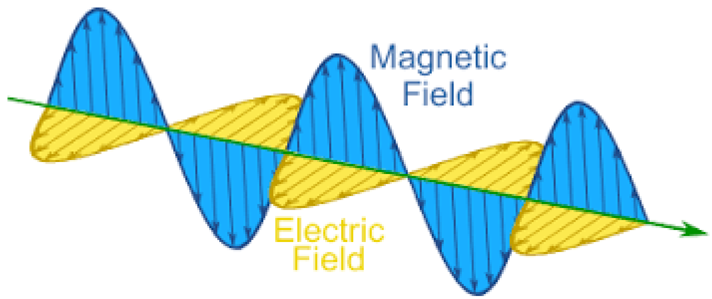
Module 2

ELECTROMAGNETISM









Electromagnetism

Module 2: Electromagnetics

- Gauss' law and its applications, Electrostatic potential, Electric fields in matter,
- Electric polarization, Bound charges, Electric Permittivity and dielectric constant. Biot-Savart law,
- Ampere's law and applications, Magnetic fields in matter, Magnetization, Bound currents, Faraday's
- law of electromagnetic induction. Displacement current and the generalized Ampere's law,
- Maxwell's equations, Electromagnetic waves

Text Book

Introduction to Electrodymanics by D J Griffiths

Introduction to electromagnetics

- Electromagnetism is a branch of Physics that deals with the force in between charged particles. This force is combination of electric and magnetic forces, therefore, called Electromagnetic force.
- Electromagnetism can be thought of as a combination of electrostatics and magnetism.
- The electromagnetic force can be attractive or repulsive.
- The electromagnetic force is one of the four fundamental forces in nature, namely,
  (i) Electromagnetic force,
  (ii) Gravitational force (weakest but has infinite range),
  (iii) Strong nuclear force (binds quarks, responsible for bound structure of nuclei),
  (iv) Weak nuclear force (radioactive decay, nuclear fission and fussion)

  Any other forces can be derived from these four forces
- The application of electromagnetics is vast. A few examples are, Optics, Antennas, Motors, Transformaer, Radar,...

Electric force

- Electric + Magnetic force = Electromagnetic force
- What is electric force?

  The force acting in between two (point) charges. It is given by Coulomb's law,

$$F = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1 q_2}{r^2}\right) = k\,\frac{q_1 q_2}{r^2} \qquad \vec{F}_{12} = \frac{1}{4\pi\varepsilon_o}\frac{q_1 q_2}{r_{21}^2}\hat{r}_{21}$$

$$k \approx 9\times 10^9\ N\,m^2 / C^2 \qquad \varepsilon_0 = 8.85\times 10^{-12}\ \mathrm{C^2/N\cdot m^2}.$$

- According to Coulomb, the electric force for charges at rest has the following properties:

  (1) Like charges repel each other, and unlike charges attract. Thus, two negative charges repel one another, while a positive charge attracts a negative charge.

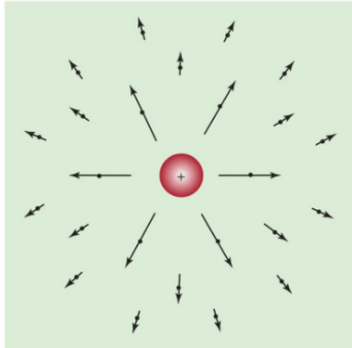
  (2) The attraction or repulsion acts along the line between the two charges.

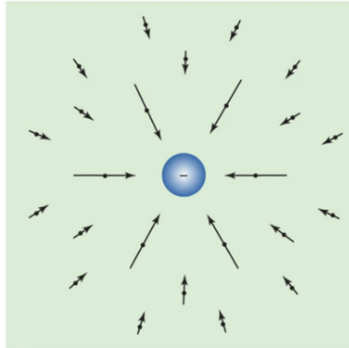
  (3) The size of the force varies inversely as the square of the distance between the two charges. Therefore, if the distance between the two charges is doubled, the attraction or repulsion becomes weaker, decreasing to one-fourth of the original value.

  (4) The size of the force is proportional to the value of each charge.

Electric field

- Every charged object sets up an electric field in the surrounding space. A second charge "feels" the presence of this field.
- The **electric field** from a charge is directed away from the charge when the charge is positive and toward the charge when it is negative.





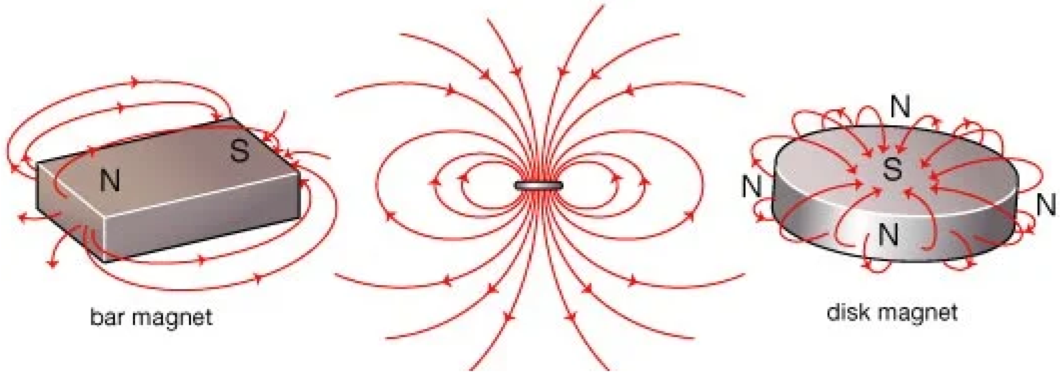
- The **electric potential** is another useful quantity. It provides an alternative to the electric field in electrostatics problems. The potential is easier to use, however, because it is a single number, a scalar, instead of a vector. The difference in potential between two places measures the degree to which charges are influenced to move from one place to another.
- The relation in between the electric field and potential is given by

$$\vec{E} = -\vec{\nabla}V \qquad \vec{E} = -\left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right)V$$

- The **electric force** can be given in terms of the electric field in the equation: $\boldsymbol{F} = q\,\boldsymbol{E}$

Magnetic field

- **Magnetic field**, a vector field in the neighbourhood of a magnet, moving electric charge, or changing electric field, in which magnetic forces are observable.



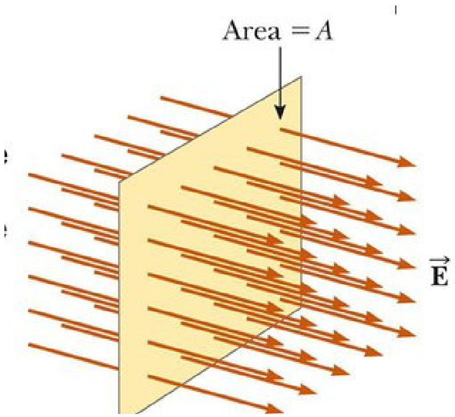
- **Magnetic force**, attraction or repulsion that arises between electrically charged particles because of their motion or in between two magnets.

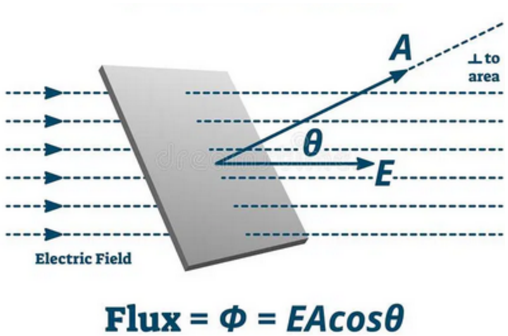
  Magnetic force acting on a moving charge *q* with velocity ***v*** is given by F = q v x B

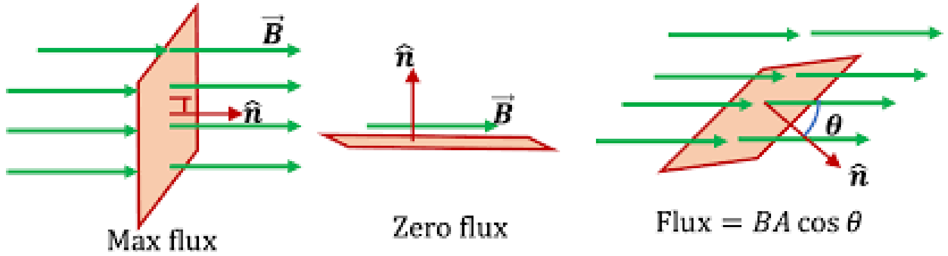
Flux

- The **electric flux** is a property of an electric field that may be thought of as the number of electric lines of force (or electric field lines) that intersect a given area.
- Electric field lines are considered to originate on positive electric charges and to terminate on negative charges.

- Magnetic fields may be represented by continuous lines of force or **magnetic flux** that emerge from north-seeking magnetic poles and enter south-seeking magnetic poles. The density of the lines indicates the magnitude of the magnetic field.
- The lines of flux are continuous, forming closed loops.







- $\Phi_E = EA$

Gauss' law

- Gauss' law states that the total electric flux passing through any closed surface is equal to the net charge *q* enclosed by it divided by ε_0.

  For a closed surface, $\phi = \oint_S \mathbf{E} \cdot d\,\mathbf{S} = \frac{q}{\varepsilon_0}$

- Here the term q on the right side of Gauss's law includes the sum of all charges enclosed by surface. The charges may be located anywhere inside the surface.

Recap - I

- **Electromagnetism** is a branch of Physics that deals with the force in between charged particles. This force is combination of electric and magnetic forces, therefore, called Electromagnetic force.

  The force acting in between two (point) charges. It is given by Coulomb's law,

  $$F = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1 q_2}{r^2}\right) = k\frac{q_1 q_2}{r^2}$$

- **Electric field** is a vector quntity around every charged object. A second charge "feels" the presence of this field.

  Magnetic field is a vector quntity around every magnetic object. A second magnetic object "feels" the presence of this field.

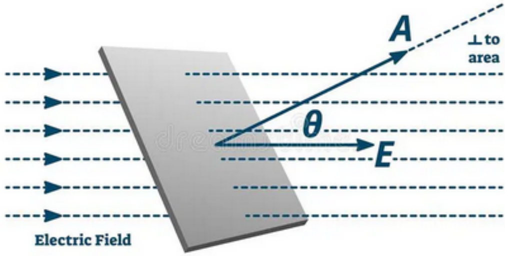
- The **electric potential** is a scalar quantity, defined as work done to bring a charge from infinity to a given point.
- The relation in between the electric field and potential is given by

  $$\vec{E} = -\vec{\nabla}V \qquad \vec{E} = -\left(\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right)V$$

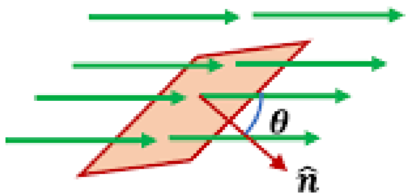
- The **electric force** is given in terms of the electric field by the equation: ***F*** = *q* ***E***
- ***Magnetic force acting on a moving charge q with velocity v*** is given by F = q ***v*** x ***B***

Recap - II

- The **electric flux** is a property of an electric field that may be thought of as the number of electric lines of force (or electric field lines) that intersect a given area. Similarly one can define magnetic flux.
- For a field ***F***, the flux *φ* through a surface is given by φ = ***F.A***, where ***A*** is the area vecto



Flux = *Φ* = *EAcosθ*



Flux $= BA\cos\theta$

- **Gauss' law** states that the total electric flux passing through any closed surface is equal to the net charge *q* enclosed by it divided by ε_0.

For a closed surface, $\phi = \oint_S \mathbf{E} \cdot d\,\mathbf{S} = \dfrac{q}{\varepsilon_0}$

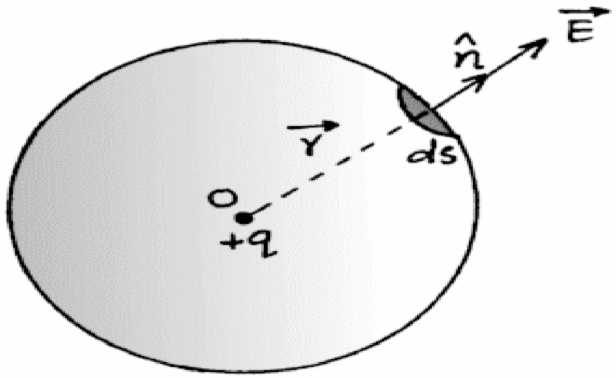
Here the term q on the right side of Gauss's law includes the sum of all charges enclosed by surface. The charges may be located anywhere inside the surface.

Proof of Gauss' law

Suppose an isolated positive point charge q is situated at the centre O of a sphere of radius r.

The electric field intensity at any point P on the surface of the sphere is

$$\vec{E} = \frac{Kq}{r^2}\hat{r}$$



where $\hat{r}$ is unit vector directed from O to P.

Total electric flux over the entire surface of sphere

$$\phi = \oint_s \vec{E}\cdot\vec{ds} = \oint_s \frac{Kq}{r^2} \times ds\,\cos 0^\circ$$

$$= \frac{Kq}{r^2}\oint_s ds$$

$$\phi = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2} \times 4\pi r^2 = \frac{q}{\epsilon_0} \qquad \left(\because \oint_s ds = 4\pi r^2\right)$$

Hence

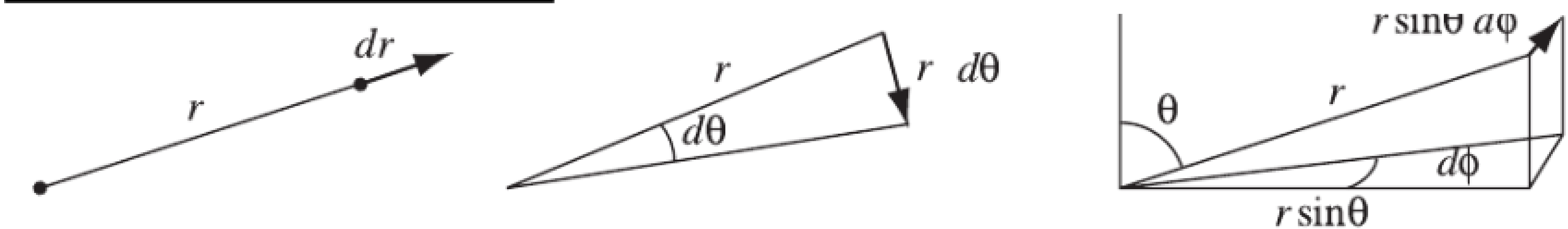
$$\phi = \oint_s \vec{E}\cdot\vec{ds} = \frac{q}{\epsilon_0}$$

Surface integral

Elemental area in Spherical-polar

$$d\vec{A} = dA\hat{r} = \left(r^2 \sin\theta d\theta d\varphi\right)\hat{r}$$

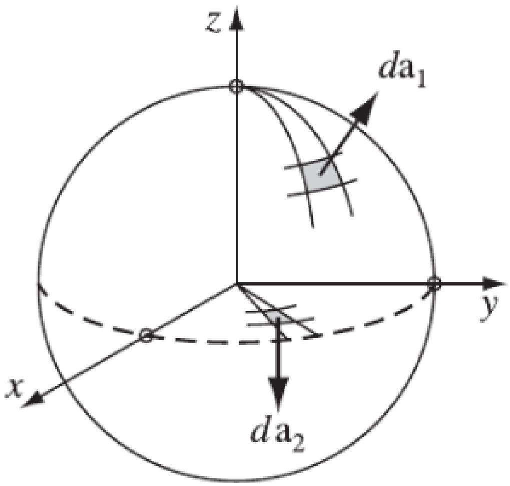
Infinitesimal Displacement:



$dl_r = dr$ $\quad$ $dl_\theta = r\, d\theta$ $\quad$ $dl_\phi = r \sin\theta\, d\phi$

Thus the general infinitesimal displacement $d\mathbf{l}$ is

$$d\mathbf{l} = dr\,\hat{\mathbf{r}} + r\,d\theta\,\hat{\boldsymbol{\theta}} + r\sin\theta\,d\phi\,\hat{\boldsymbol{\phi}}.$$



If you are integrating on the surface of a sphere,

$$d\mathbf{a}_1 = dl_\theta\,dl_\phi\,\hat{\mathbf{r}} = r^2\sin\theta\,d\theta\,d\phi\,\hat{\mathbf{r}}.$$

Proof of Gauss' law

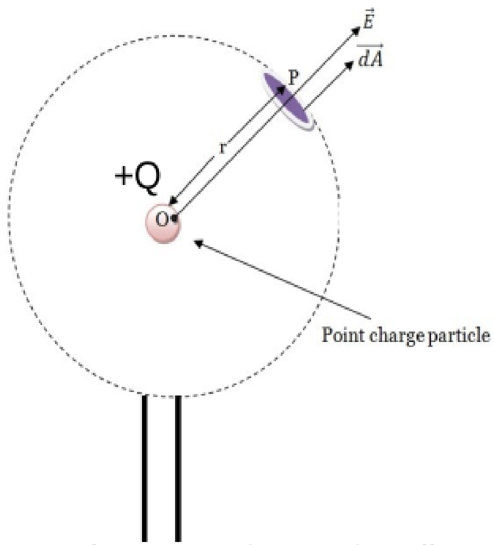
Elemental area in Spherical-polar $d\vec{A} = dA\hat{r} = \left(r^2 \sin\theta d\theta d\varphi\right)\hat{r}$

The total Flux through the closed Gaussian surface:

$$\Phi_E = \oint_S \vec{E}(\vec{r})\bullet d\vec{A} = \frac{Q}{4\pi\varepsilon_o}\int_S \left(\frac{1}{r^2}\hat{r}\right)\bullet\underbrace{\left(r^2 \sin\theta d\theta d\varphi \hat{r}\right)}_{=d\vec{A}}$$

$$\text{Thus:}\quad \Phi_E = \frac{Q}{4\pi\varepsilon_o}\int_{\theta=0}^{\theta=\pi}\int_{\varphi=0}^{\varphi=2\pi}\sin\theta d\theta d\varphi\underbrace{\left(\hat{r}\bullet\hat{r}\right)}_{=1} = \frac{2\pi Q}{\underbrace{4\pi}_{2}\varepsilon_o}\int_{\theta=0}^{\theta=\pi}\sin\theta d\theta$$

$$= \frac{2Q}{2\varepsilon_o} = \frac{Q}{\varepsilon_o}$$



Imaginary surface of radius R surrounding the charge; ***Gaussian Surface***

Gauss' Law (in Integral Form): $\Phi_E = \oint_S \vec{E}(\vec{r})\bullet d\vec{A} = \frac{Q_{enclosed}}{\varepsilon_o}$ → ***For any surface enclosing the charge***

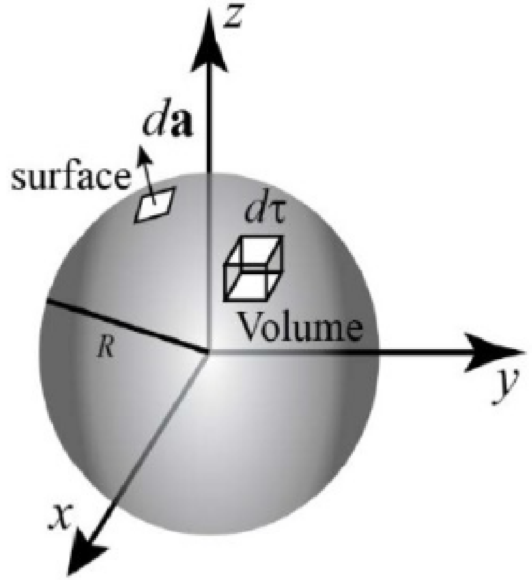
Electric flux through closed surface S = (electric charge enclosed by surface S)/ ε_o

Can we prove the above statement for arbitrary closed shape?

Gauss' divergence theorem

As it stands, Gauss's law is an *integral* equation, but we can easily turn it into a *differential* one, by applying the divergence theorem:

$$\oint_{\mathcal{S}} \mathbf{E} \cdot d\mathbf{a} = \int_{\mathcal{V}} (\nabla \cdot \mathbf{E})\, d\tau.$$



→ *For a collection of charges enclosed by a Gaussian surface:*

Rewriting Q_{enc} in terms of the charge density ρ, we have

$$Q_{\text{enc}} = \int_{\mathcal{V}} \rho\, d\tau.$$

So Gauss's law becomes

$$\int_{\mathcal{V}} (\nabla \cdot \mathbf{E})\, d\tau = \int_{\mathcal{V}} \left(\frac{\rho}{\epsilon_0}\right) d\tau.$$ *(Holds for any volume)*

Therefore,

$$\boxed{\nabla \cdot \mathbf{E} = \frac{1}{\epsilon_0}\rho.}$$

→ *Gauss' Law in differential form*

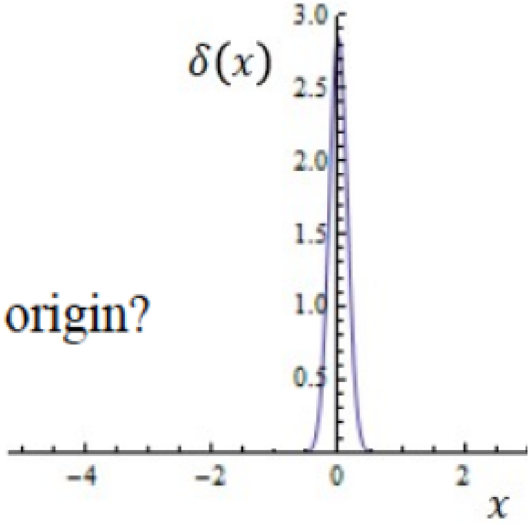
Gauss' divergence theorem

Gauss Theorem States that – If V is the volume bound by the surface S, volume integral of divergence of a function $\vec{v}$ over volume V is equal to surface integral of the function $\vec{v}$ over the surface S that surrounds the given volume.

Dirac delta function

- Dirac delta function is a special function, which is defined as:

$$\delta(x) = 0, \quad if\ x \neq 0$$
$$= \infty, \quad if\ x = 0$$
$$\int_{-\infty}^{\infty} \delta(x)dx = 1$$



- Example: What is the charge density of a point charge q kept at the origin?

$$\rho(x) = q\delta(x); \qquad \int_{-\infty}^{\infty} \rho(x)dx = \int_{-\infty}^{\infty} q\delta(x)dx = q$$

→ If $f(x)$ is a continuous function of x

$$\int_{-\infty}^{\infty} f(x)\delta(x-a)dx = \int_{-\infty}^{\infty} f(a)\delta(x-a)dx = f(a)$$

→ 3D Dirac delta function is defined as:

$$\delta^3(\mathbf{r}) = \delta(x)\delta(y)\delta(z) \qquad \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(\mathbf{r})\delta^3(\mathbf{r}-\mathbf{a}) = f(\mathbf{a})$$

Divergence of electric field

Let's go back, now, and calculate the divergence of $\mathbf{E}$ directly

$$\mathbf{E}(\mathbf{r}) = \frac{1}{4\pi\epsilon_0} \int_{\text{all space}} \frac{\hat{\boldsymbol{\mathscr{r}}}}{\mathscr{r}^2} \rho(\mathbf{r}')\, d\tau'.$$

Then, $\boldsymbol{\nabla} \cdot \mathbf{E} = \frac{1}{4\pi\epsilon_0} \int \boldsymbol{\nabla} \cdot \left(\frac{\hat{\boldsymbol{\mathscr{r}}}}{\mathscr{r}^2}\right) \rho(\mathbf{r}')\, d\tau'$

If, $\mathbf{V} = \frac{\hat{\mathbf{r}}}{r^2}$

$$\boldsymbol{\nabla} \cdot \mathbf{V} = \boldsymbol{\nabla} \cdot \left(\frac{\hat{\mathbf{r}}}{r^2}\right) = \frac{0}{r^3} = 0$$

Except at r=0, where it reduces to 0/0; not defined

$$\boldsymbol{\nabla} \cdot \left(\frac{\hat{\boldsymbol{\mathscr{r}}}}{\mathscr{r}^2}\right) = 4\pi\delta^3(\boldsymbol{\mathscr{r}}).$$

→ Dirac delta function, non-zero only at r=0

Thus,

$$\boldsymbol{\nabla} \cdot \mathbf{E} = \frac{1}{4\pi\epsilon_0} \int 4\pi\delta^3(\mathbf{r} - \mathbf{r}')\rho(\mathbf{r}')\, d\tau' = \frac{1}{\epsilon_0}\rho(\mathbf{r})$$

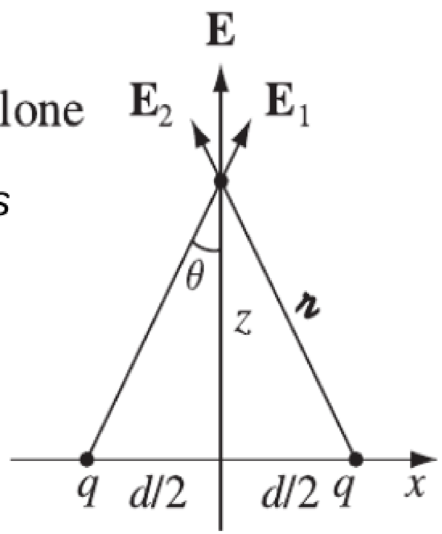
Hence,

$$\int_{\mathcal{V}} \boldsymbol{\nabla} \cdot \mathbf{E}\, d\tau = \oint_{\mathcal{S}} \mathbf{E} \cdot d\mathbf{a} = \frac{1}{\epsilon_0} \int_{\mathcal{V}} \rho\, d\tau = \frac{1}{\epsilon_0} Q_{\text{enc}}.$$

Application of Gauss' law:
To find electric field and potential due to a multiple point charges or charge distribution.

Calculating electric field: two point charges (++)

Find the electric field a distance *z* above the midpoint between two equal charges (*q*), a distance *d* apart.



→ Let $\mathbf{E}_1$ be the field of the left charge alone, and $\mathbf{E}_2$ that of the right charge alone

Vectorially adding them the horizontal components cancel and the vertical components add up.

$$E_z = 2\frac{1}{4\pi\epsilon_0}\frac{q}{\mathscr{r}^2}\cos\theta.$$

Here $\mathscr{r} = \sqrt{z^2 + (d/2)^2}$ and $\cos\theta = z/\mathscr{r}$, so

$$\mathbf{E} = \frac{1}{4\pi\epsilon_0}\frac{2qz}{\left[z^2 + (d/2)^2\right]^{3/2}}\hat{\mathbf{z}}.$$

Check: When $z \gg d$ you're so far away that it just looks like a single charge $2q$, so the field should reduce to $\mathbf{E} = \frac{1}{4\pi\epsilon_0}\frac{2q}{z^2}\hat{\mathbf{z}}$. And it *does* (just set $d \to 0$ in the formula).