What if Electric/Magnetic fields are not constant?

- Put a static charge is (constant) electric field, it feels the electrostatis force, ***F***= *q**E***.
- Due to the force on charge, it starts moving, producing a current *I = dq/dt*
- Biot-Savart law says that a (steady) current produces a (constant) magnetic field.
- If a moving charge or current carrying material is placed under an external magnetic field, the charge feels Lorentz force perpendicular to both magnetic field and charge flow direction. ***F*** = *q* (***v*** x ***B***).
- If the magnetic field is not constant, the changing magnetic flux induces electromotive force (emf), or potential drop (or extra electric field), the current carrying object !
- Maxwell's equations of electromagnetism:

  Maxwell's 3rd and 4th equations show the interdependence of electric field and magnetic field.

$$\nabla \cdot \mathbf{E} = \frac{1}{\epsilon_0}\rho$$

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\nabla \times \mathbf{B} = \mu_0 \mathbf{J} + \frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t}$$

Faraday's law of induction

- The amount of induced emf is given by the **Faraday's law of induction**:

  First Law : The changing magnetic field linked with a conductor induces an electromotive force in the conductor.

  Second Law : The induced electromotive force is proportional to the rate of change of magnetic field linked with the conductor.

$$\mathcal{E} = -\frac{\mathrm{d}\Phi_B}{\mathrm{d}t}$$

Faraday's law of induction: Proof

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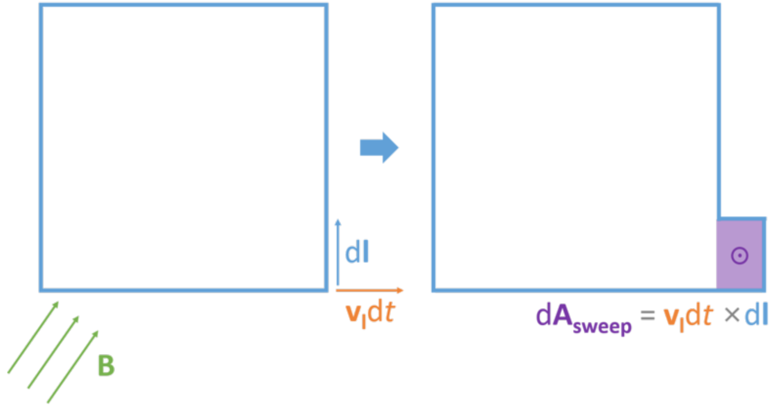

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\int_S (\vec{\nabla} \times \vec{E}).d\vec{S} = -\int_S \frac{\partial \vec{B}}{\partial t}.d\vec{S}$$

$$\oint \vec{E}.d\vec{l} = -\int_S \frac{\partial \vec{B}}{\partial t}.d\vec{S} = -\frac{d}{dt}\int_S \vec{B}.d\vec{S}$$

$$\oint \vec{E}.d\vec{l} = -\frac{d\phi_B}{dt}$$

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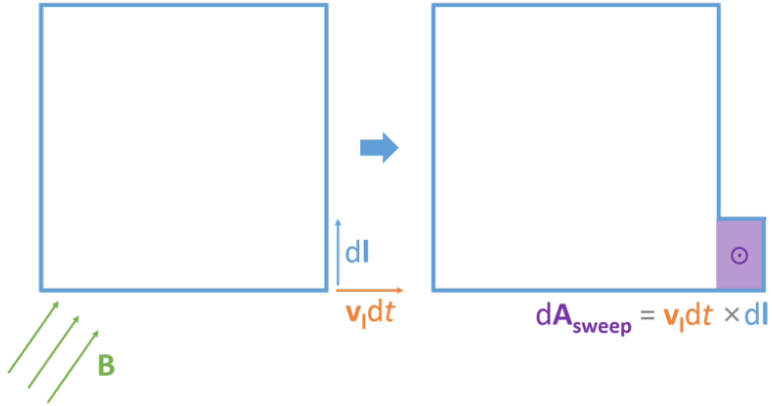
$$\frac{d\phi_B}{dt} = \frac{d}{dt}\int_S \vec{B}.\,d\vec{S}$$
$$= \int_{S(t_0)} \frac{\partial B\vec{(t)}}{dt}\,\partial\vec{S} + \frac{d}{dt}\int_{S(t)} B\vec{(t_0)}.\,d\vec{S}$$
$$= -\oint \vec{E}(t_0)\,.\,d\vec{l} + \frac{d}{dt}\int_{S(t)} B\vec{(t_0)}.\,d\vec{S}$$



Faraday’s law of induction

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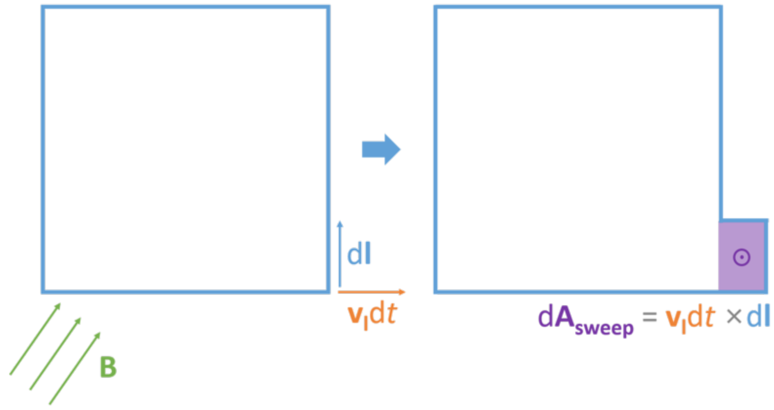

$$d\phi_B = \int_{S(t)} \vec{B}(t_0).(\vec{v}_l dt \times d\vec{l})\, d\vec{S} = -\int_{S(t)} dt d\vec{l}.(\vec{v}_l \times \vec{B})$$

$$\frac{d\phi_B}{dt} = -\oint \vec{E}(t_0)\, .\, d\vec{l} + \frac{d}{dt}\int_{S(t)} B(\vec{t}_0).\, d\vec{S}$$
$$= -\oint \vec{E}(t_0)\, .\, d\vec{l} - \oint (\vec{v}_l(t_0 \times B(\vec{t}_0).\, d\vec{l}$$
$$= -\oint [E(t_0) + (\vec{v}_l(t_0) \times \vec{B}(t_0))].d\vec{l}$$

$$\mathcal{E} = \oint (\vec{E} + \vec{v} \times \vec{B}).d\vec{l}$$