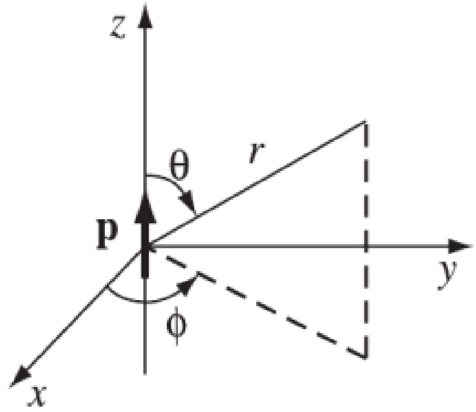
Electric field due to dipole

We have found out the potential in terms of dipole moment:

$$V_{\text{dip}}(r,\theta) = \frac{\hat{\mathbf{r}}\cdot\mathbf{p}}{4\pi\epsilon_0 r^2} = \frac{p\cos\theta}{4\pi\epsilon_0 r^2}$$

To get the field, we take the negative gradient of V:



$$E_r = -\frac{\partial V}{\partial r} = \frac{2p\cos\theta}{4\pi\epsilon_0 r^3},$$

$$E_\theta = -\frac{1}{r}\frac{\partial V}{\partial\theta} = \frac{p\sin\theta}{4\pi\epsilon_0 r^3},$$

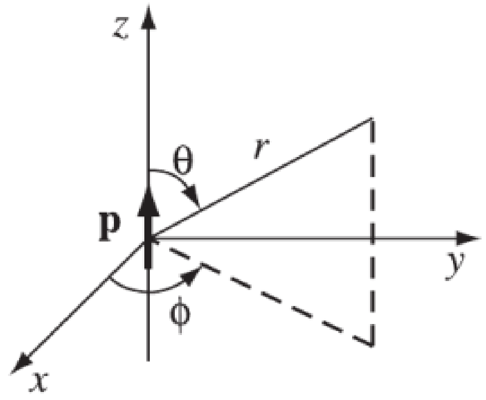
$$E_\phi = -\frac{1}{r\sin\theta}\frac{\partial V}{\partial\phi} = 0.$$

Thus,

$$\mathbf{E}_{\text{dip}}(r,\theta) = \frac{p}{4\pi\epsilon_0 r^3}(2\cos\theta\,\hat{\mathbf{r}} + \sin\theta\,\hat{\boldsymbol{\theta}}).$$

Electric field due to dipole

The electric field can be rewritten as



$$\mathbf{E}_{\text{dip}}(\mathbf{r}) = \frac{1}{4\pi\epsilon_0 r^3}[3(\mathbf{p}.\hat{\mathbf{r}})\hat{\mathbf{r}} - \mathbf{p}]$$

Since,

$$\mathbf{p} = (\mathbf{p}.\hat{\mathbf{r}})\ \hat{\mathbf{r}} + (\mathbf{p}.\hat{\theta})\ \hat{\theta}$$
$$= p\cos\theta\ \hat{\mathbf{r}} - p\sin\theta\ \hat{\theta}$$

Then,

$$3(\mathbf{p}.\hat{\mathbf{r}})\ \hat{\mathbf{r}} - \mathbf{p}$$
$$= 3p\cos\theta\ \hat{\mathbf{r}} - p\cos\theta\ \hat{\mathbf{r}} + p\sin\theta\ \hat{\theta}$$
$$= 2p\cos\theta\ \hat{\mathbf{r}} + p\sin\theta\ \hat{\theta}$$

Dielectrics

Almost all everyday objects upto a good approximation, belong to one of the two large categories: **conductors** and **insulators** or **dielectrics**.

→ Conductors have an unlimited supply of free electrons that move through the material.

→ In dielectrics all charges are attached to specific atoms or molecules. They can move a little bit within the atom or molecule but the microscopic displacements are not as significant as inside a conductor. The cumulative effect of these microscopic movements accounts for the characteristics of a dielectric.

→ Depending on the amount of restriction on the movement of the charges inside the material we can differentiate between insulators and dielectrics. Insulators have the smallest dielectric constant.

Atomic dipole

When neutral atom is placed in an electric field:

→ The positive and negative charge cores are separated; the nucleus pushed in the direction of the field and the electrons the opposite way.

→ The positive and negative core attract each other; it holds the atom together.

→ The atom is polarised and now has a tiny dipole moment pointing in the same direction as the external electric field.

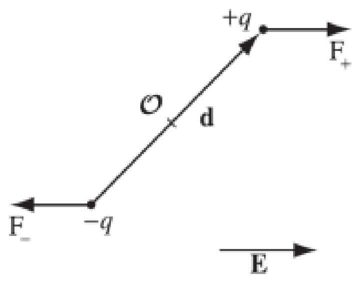
$$\mathbf{p} = \alpha \mathbf{E}.$$

The constant of proportionality α is called **atomic polarizability.**

Some molecules have built-in permanent dipole moment. When such polar molecules are placed in an electric field:

→ If the field is uniform, the force on the positive end exactly cancels the force on the negative end.

→ There is a torque.



$$\mathbf{N} = (\mathbf{r}_+ \times \mathbf{F}_+) + (\mathbf{r}_- \times \mathbf{F}_-)$$
$$= \left[(\mathbf{d}/2) \times (q\mathbf{E})\right] + \left[(-\mathbf{d}/2) \times (-q\mathbf{E})\right] = q\mathbf{d} \times \mathbf{E}.$$

Hence, we can write,

$$\mathbf{N} = \mathbf{p} \times \mathbf{E}.$$

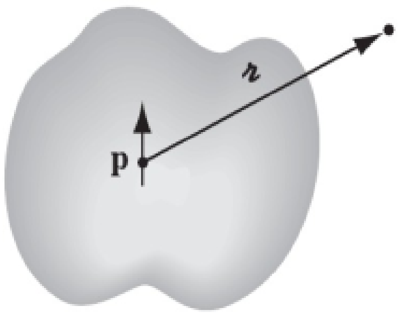
Polarization

Notice that these two mechanisms produce the same basic result: *a lot of little dipoles pointing along the direction of the field*—the material becomes **polarized**. A convenient measure of this effect is

$$\mathbf{P} \equiv \textit{dipole moment per unit volume},$$

which is called the **polarization**.

Say, we have a polarized material with all the dipoles pointing in the same diection.



For a single dipole, the electrostatic potential can be written as

$$V(\mathbf{r}) = \frac{1}{4\pi\epsilon_0}\frac{\mathbf{p}\cdot\hat{\boldsymbol{\mathscr{r}}}}{\mathscr{r}^2}$$

where, $\boldsymbol{\mathscr{r}}$ is the vector from the dipole to the point at which we are trying to find the potential. We have a dipole moment $\mathbf{p} = \mathbf{P}\,d\tau'$ in each volume element $d\tau'$. Hence the total potential is

$$V(\mathbf{r}) = \frac{1}{4\pi\epsilon_0}\int_{\mathcal{V}} \frac{\mathbf{P}(\mathbf{r}')\cdot\hat{\boldsymbol{\mathscr{r}}}}{\mathscr{r}^2}\,d\tau'$$

Field due to polarized object

The total potential can be written as,

$$V = \frac{1}{4\pi\epsilon_0}\int_{\mathcal{V}} \mathbf{P}\cdot\nabla'\left(\frac{1}{\mathscr{r}}\right)d\tau' \qquad \text{since,} \qquad \nabla'\left(\frac{1}{\mathscr{r}}\right) = \frac{\hat{\boldsymbol{\mathscr{r}}}}{\mathscr{r}^2}$$

Using the identity $\nabla\cdot(f\mathbf{A}) = f(\nabla\cdot\mathbf{A}) + \mathbf{A}\cdot(\nabla f)$

$$V = \frac{1}{4\pi\epsilon_0}\left[\int_{\mathcal{V}} \nabla'\cdot\left(\frac{\mathbf{P}}{\mathscr{r}}\right)d\tau' - \int_{\mathcal{V}} \frac{1}{\mathscr{r}}(\nabla'\cdot\mathbf{P})\,d\tau'\right]$$

Using divergence theorem,

$$V = \frac{1}{4\pi\epsilon_0}\oint_{\mathcal{S}} \frac{1}{\mathscr{r}}\mathbf{P}\cdot d\mathbf{a}' - \frac{1}{4\pi\epsilon_0}\int_{\mathcal{V}} \frac{1}{\mathscr{r}}(\nabla'\cdot\mathbf{P})\,d\tau'$$

Potential due to a surface charge distribution

$$\sigma_b \equiv \mathbf{P}\cdot\hat{\mathbf{n}}$$

Potential due to a volume charge distribution

$$\rho_b \equiv -\nabla\cdot\mathbf{P}$$

Bound charges

Potential of a polarized object is the same as that produced by a volume and a surface charge distribution.

Bound charges

$$\sigma_b \equiv \mathbf{P} \cdot \hat{\mathbf{n}}$$

$$\rho_b \equiv -\nabla \cdot \mathbf{P}$$

$$V(\mathbf{r}) = \frac{1}{4\pi\epsilon_0} \oint_{\mathcal{S}} \frac{\sigma_b}{\mathscr{r}} \, da' + \frac{1}{4\pi\epsilon_0} \int_{\mathcal{V}} \frac{\rho_b}{\mathscr{r}} \, d\tau'$$

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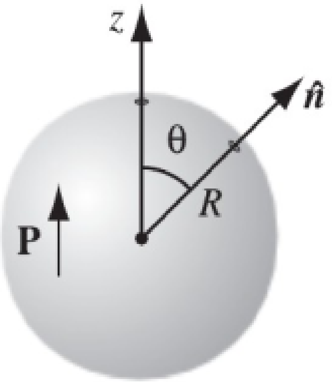
Ex: Find the electric field produced by a uniformly polarized sphere of radius R



→ The volume bound charge density ρ_b is zero, since $\mathbf{P}$ is uniform

→ choose the z axis to coincide with the direction of polarization

$$\sigma_b = \mathbf{P} \cdot \hat{\mathbf{n}} = P\cos\theta$$

→ charge density $P\cos\theta$ plastered over the surface of a sphere