Tutorial Set -

Solution Set 2

(1) $V = \frac{5}{r^2}\cos\theta$

$$\vec{E} = -\vec{\nabla} V$$

$$= -\left(\frac{\partial V}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial V}{\partial \theta}\hat{\theta}\right)$$

$$= -\left[-2\frac{5}{r^3}\cos\theta\,\hat{r} + \frac{1}{r}\frac{5}{r^2}(-\sin\theta)\hat{\theta}\right]$$

$$= \frac{10}{r^3}\cos\theta\,\hat{r} + \frac{5}{r^3}\sin\theta\,\hat{\theta}$$

At $r = 2,\ \theta = \frac{\pi}{2}$ & $\phi = 0$,

$$\vec{E} = \frac{10}{8}\cos\frac{\pi}{2}\,\hat{r} + \frac{5}{8}\sin\frac{\pi}{2}\,\hat{\theta}$$

$$= \frac{5}{8}\hat{\theta}$$

Q1

The field inside the conductor will be equal in magnitude to the external field while its direction will be opposite. Hence, the electric field inside

$$\vec{E} = -E_0\hat{k} = -\left(\frac{\sigma}{\epsilon_0}\right)\hat{k}$$

$$\Rightarrow E_0 = \frac{\sigma}{\epsilon_0}$$

$$\Rightarrow \sigma = \epsilon_0 E_0$$

[Here, we assume that the parallel surfaces are of infinite length and width]

Q2

$$\vec{F}_1 = x^2\hat{i} + 3xz^2\hat{j} - 2xz\hat{k}$$

For this vector to represent an electrostatic field, we need

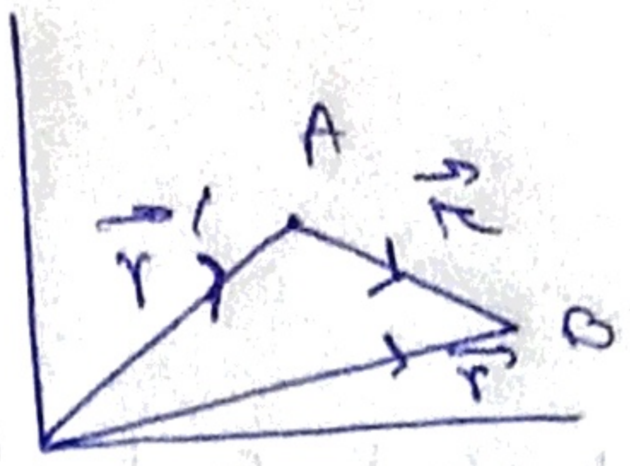
$$\vec{\nabla}\times\vec{F}_1 = 0$$

$$\vec{\nabla}\times\vec{F}_1 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x^2 & 3xz^2 & -2xz \end{vmatrix} = \hat{i}\,(0 - 3x(2z)) + \hat{j}\,(0 + 2z) + \hat{k}\,(3z^2 - 0)$$

$$= -6xz\,\hat{i} + 2z\,\hat{j} + 3z^2\,\hat{k}$$

Hence, $\vec{F}_1$ cannot represent an electrostatic field.

Q3



We know,

$$\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{q}{R^2}\hat{R}$$

$$\vec{r}' + \vec{R} = \vec{r}$$

$$\Rightarrow \vec{R} = \vec{r} - \vec{r}'$$

Here,

$$A \equiv (3, 2, 0)$$

$$B \equiv (3, 5, 0)$$

$$\Rightarrow \vec{R} = (3-3)\hat{i} + (5-2)\hat{j} + (0-0)\hat{k}$$

$$= 3\hat{j}$$

Hence, $\vec{E} = \frac{1}{4\pi\epsilon_0}\frac{1\times10^{-6}}{9}\hat{j}$

$$= \frac{10^{-6}}{36\pi\epsilon_0}\hat{j} \quad \text{(in SI unit)}$$

(5) $\rho(r) = \rho_0\left(1 - \frac{4r}{3R}\right) \qquad 0 < r < R$

$= 0 \qquad r > R$

For $r < R$

Total charge enclosed

$$q = \rho_0\int\left(1 - \frac{4r'}{3R}\right)r'^2\sin\theta\, d\theta\, d\phi\, dr'$$

$$= \rho_0\int_0^r\left(1 - \frac{4r'}{3R}\right)r'^2\,dr'\int_0^\pi \sin\theta\,d\theta\int_0^{2\pi}d\phi$$

$$= 4\pi\rho_0\left[\frac{r'^3}{3} - \frac{4}{3R}\cdot\frac{r'^4}{4}\right]_0^r$$

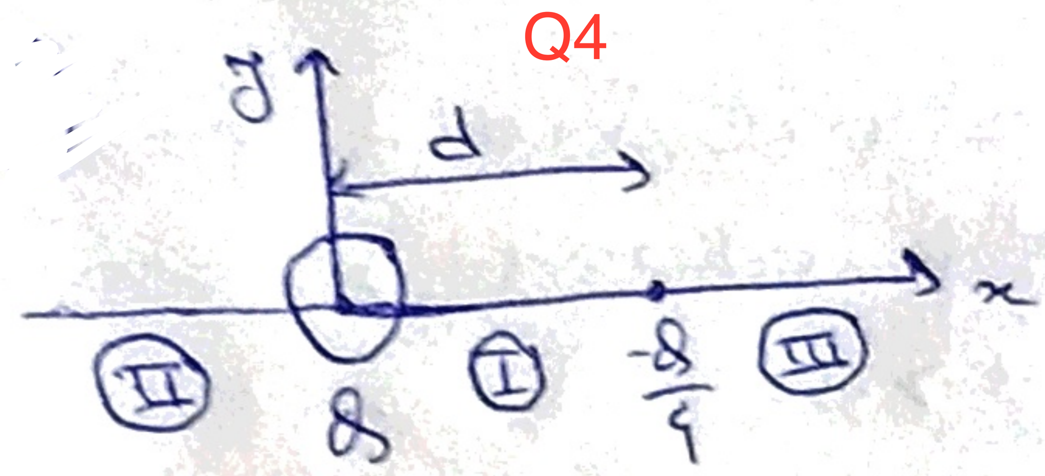
$$= 4\pi\rho_0\left[\frac{r^3}{3} - \frac{r^4}{3R}\right]$$

$$\Rightarrow \vec{E} = \frac{\rho}{\epsilon_0}\frac{\left(\frac{r^3}{3} - \frac{r^4}{3R}\right)}{r^2}\hat{r}$$

$$= \frac{\rho}{\epsilon_0}\left(\frac{r}{3} - \frac{r^2}{3R}\right)\hat{r}$$

$$= \frac{\rho}{3\epsilon_0}\left(r - \frac{r^2}{R}\right)\hat{r}$$

Q4



The net electric field can only be zero for points on the x-axis.

Let us look at the 3 regions.

Region (I)

For any points in this region, $\vec{E}_1$ (due to q) and $\vec{E}_2$ (due to $-\frac{q}{4}$) both acts on $\hat{x}$ directions. Net electric field cannot be zero.

Region (II)

For any points in this region, $\vec{E}_1$ points in $-\hat{x}$ direction and $\vec{E}_2$ in $+\hat{x}$ direction, but $|\vec{E}_1| > |\vec{E}_2|$ since $|\vec{r}_1| < |\vec{r}_2|$ and $|q| > |\frac{q}{4}|$.
Net electric field cannot be zero.

Region (III)

At $x > d$, say $x = x' + d$

$$\vec{E}_1 = \frac{q\hat{x}}{4\pi\epsilon_0 (x'+d)^2}$$

$$\vec{E}_2 = \frac{-q\hat{x}}{16\pi\epsilon_0 x'^2}$$

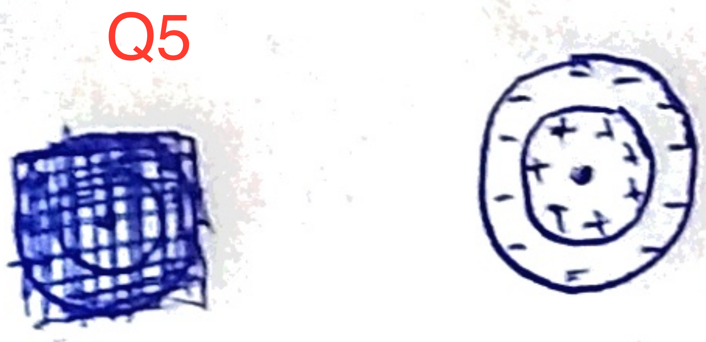
Hence, the requirement is

$$\frac{1}{(x'+d)^2} = \frac{1}{4x'^2}$$

$$\Rightarrow \quad 2x' = x' + d \Rightarrow x' = d$$

→ The net electric field can be zero at $x = 2d$

Q5



$-1\,\mu C$ is placed at the center.

→ The inner wall will have an induced charge $+1\,\mu C$ distributed uniformly

→ The outer wall will have $-1\,\mu C$ charge distributed uniformly.

Outer surface will have a charge density

$$= \frac{-10^{-6}}{4\pi}\ C.m^{-2}$$

$$= -\frac{10^{-6}}{4\pi}\ \frac{C}{m^2}$$ (Surface charge density)