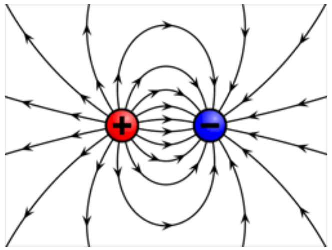
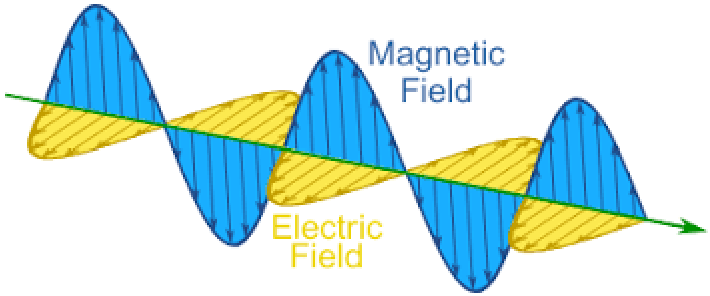
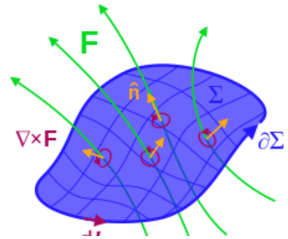
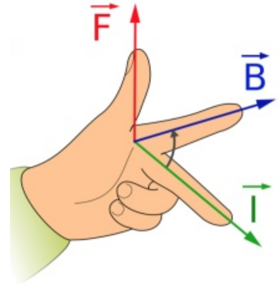
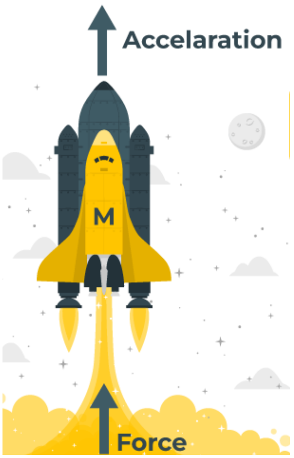


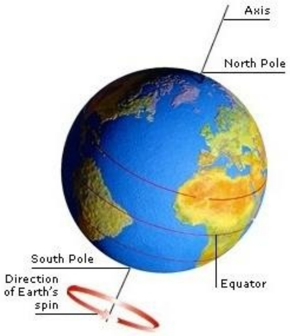


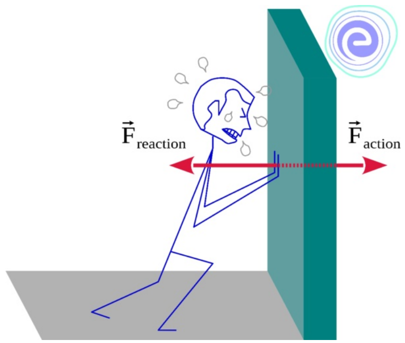




ELECTROMAGNETISM
AND
MECHANICS







Electromagnetism & Mechanics (EPHY111L)

L – T – P : 3 - 1 – 2 (5 credit) poulomi.sadhukhan@bennett.edu.in (M144)

Module 1: Mechanics

- A brief review of vectors, and coordinate systems;
  Velocity, acceleration, and kinematic equations in different coordinates systems;
- Newton's laws, momentum, work, and energy.
  Conservative and non-conservative forces, connection between force and potential energy;
- Angular Momentum and rotational motion.
  Noninertial frames of reference and pseudo forces.
- Central Force motion: planetary motion, and Kepler's laws.
- Harmonic Oscillator.

Module 2: Electromagnetics

- Electrostatic potential, Electric field, Polarization, Permittivity, Dielectric constant
- Magnetic field by current carrying conductor: Biot-Savart law, Ampere's law
- Current carrying conductor in varying magnetic field: Electromagnetic force, Faraday's law
- Introduction to Maxwell's equation

(Check LMS for detailed syllabus)

Text Book

An Introduction to Mechanics, by Daniel Kleppner and Robert Kolenkow, Second Edition, Cambridge University Press.

Reference Book

Fundamentals of Physics, by David Halliday, Robert Resnick, and Jearl Walker, John Wiley & Sons, 10th Edition.

University Physics, by Hugh D. Young and Roger A. Freedman, Pearson, 13th Edition.

(Hard copies are available in Library. E-books are available at https://www.bennett.refread.com)

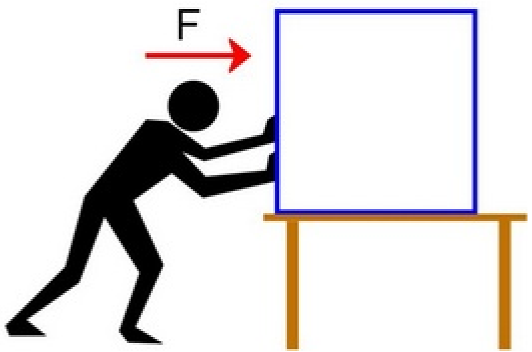
Evaluation components:

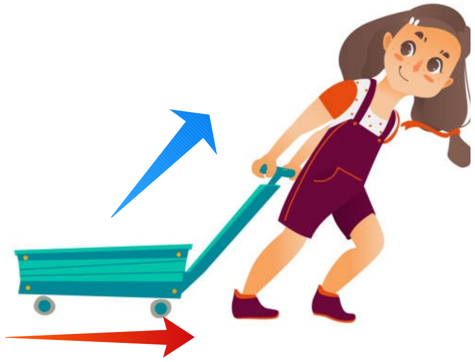
Quiz: 15 %
Midterm: 25%
Endterm: 35%
Lab: 25%

Mechanics

- What is Mechanics?

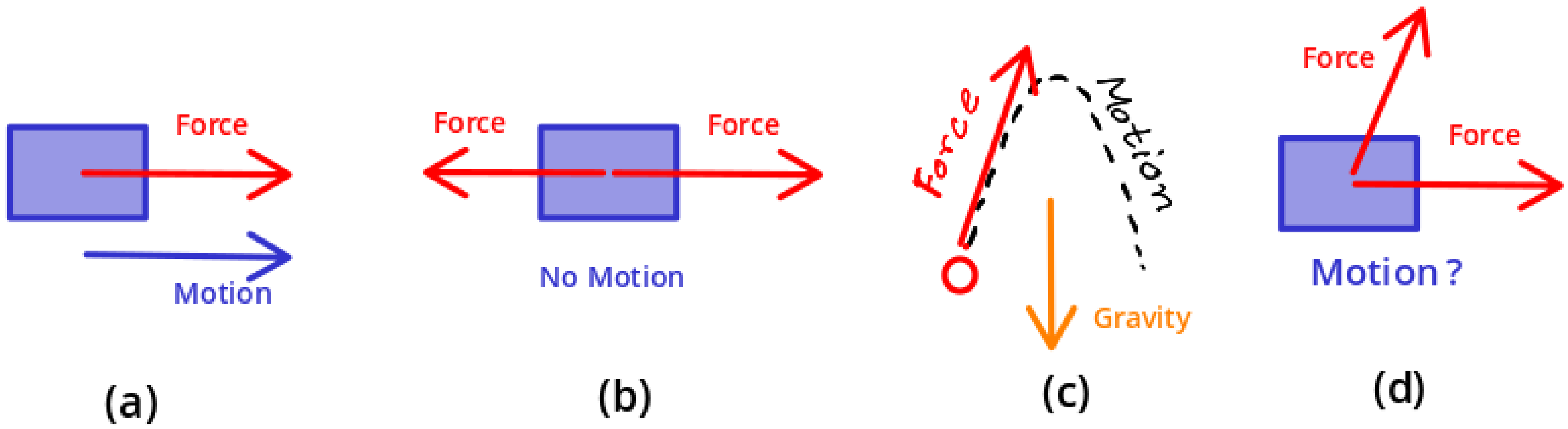
  Mechanics, branch of physics concerned with the motion of bodies under the action of **forces**, including the special case in which a body remains at rest.





Direction matters! ...and magnitude too.

Direction & magnitude: Vector



- A **vector** is a quantity that has both magnitude and direction.

  It is typically represented by an arrow whose direction is the same as that of the quantity and length is proportional to the quantity's magnitude.

  Example: Force, velocity, acceleration, torque

- In contrast to vectors, ordinary quantities that have a magnitude but **not a direction** are called **scalars**.

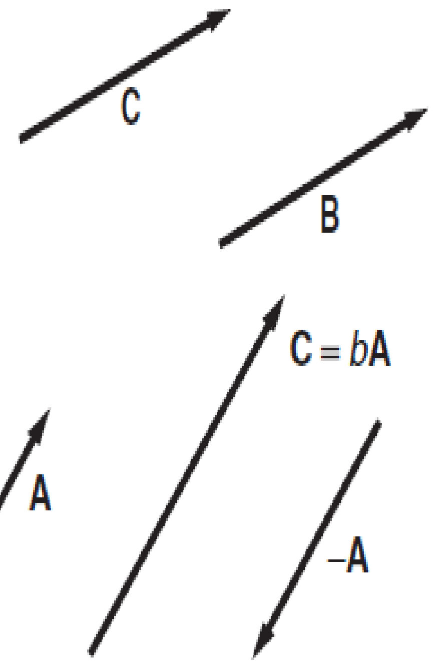
  Example: Mass, time

Vector analysis

- While scalars quantites in Physics and Mathemetics are represented simply by a letter (Ex. A, m, t, P), vectors are represented by a letter with overhead arrow to denote the direction associated with it (Ex. $\vec{F}$, $\vec{v}$, $\vec{a}$), or a bold letter: ***F, v, a***

<u>VECTOR ALGEBRA</u>

- Two vectors are equal if both the magnitude and directions are same.
- If the length (magnitude) of a vector is unity, it is called unit vector.
- If we multiply a vector with a constant (scalar), the magnitude of the vector gets multiplied.
- Multiplying by -1 flips the direction of the vector.



Coordinate system

But, how to specify direction?

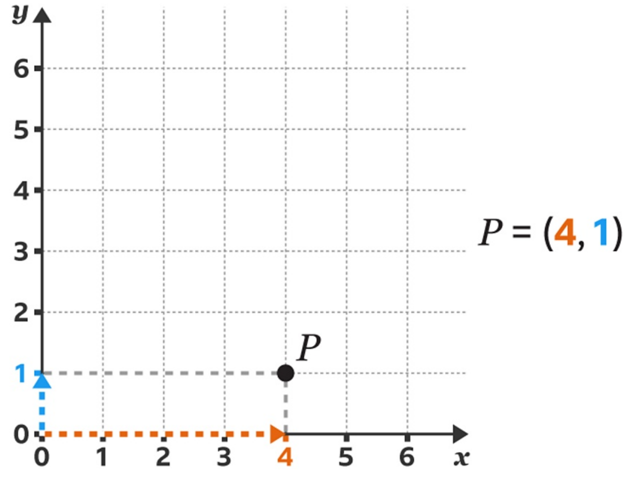
How to add, subtraction, multiply two vectors??

We need help of “Coordinate system”

Coordinate system: An arrangement of reference lines to identify the position of a point.

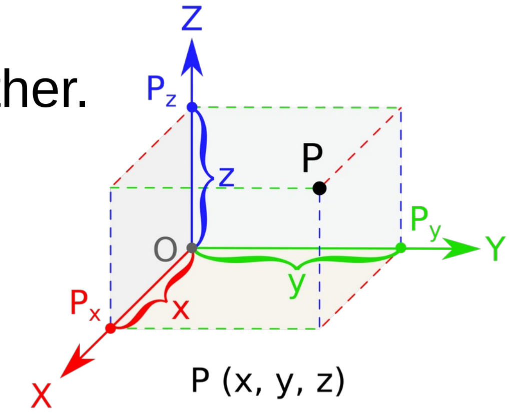
- Where is the point *P*?
  4 units along *x*-axis, 1 unit along *y*-axis.
  (components)

  Dimension of space: 2D
  So, at least two reference lines are required.



Cartesian Coordinate system

- In 3D, 3 reference lines are required to specify a point.
  The reference lines (or axis) are perpendicular to each other.

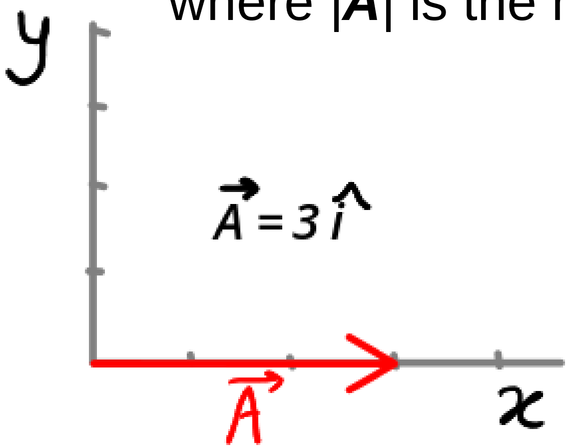


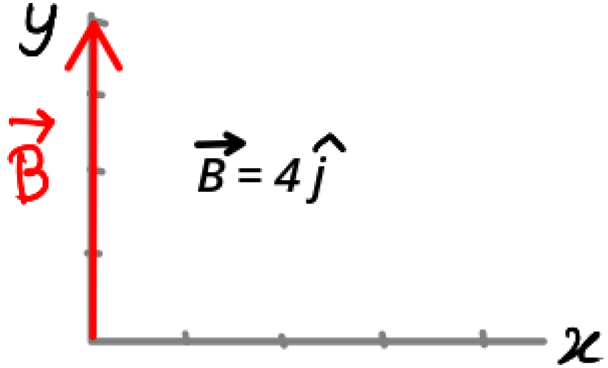
- To describe vectors, we need direction.

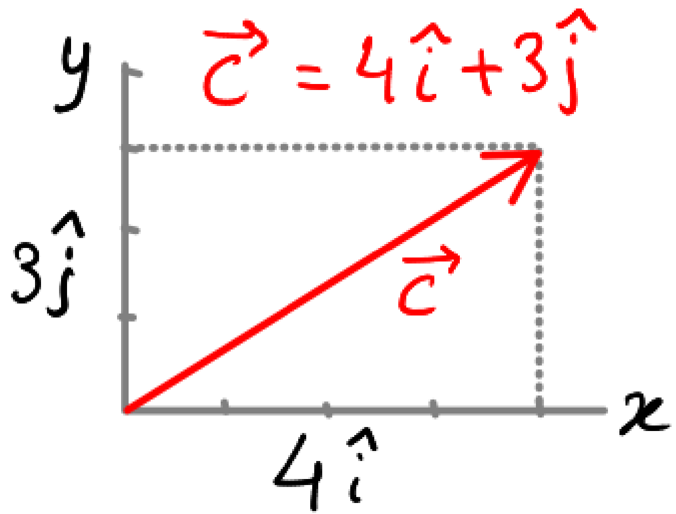
  Units of 3 different directions, *x, y,* & *z*, are denoted as

$$\hat{x}\,,\quad \hat{y}\,,\quad \hat{z}\qquad \text{or}\qquad \hat{i}\,,\quad \hat{j}\,,\quad \hat{k}$$

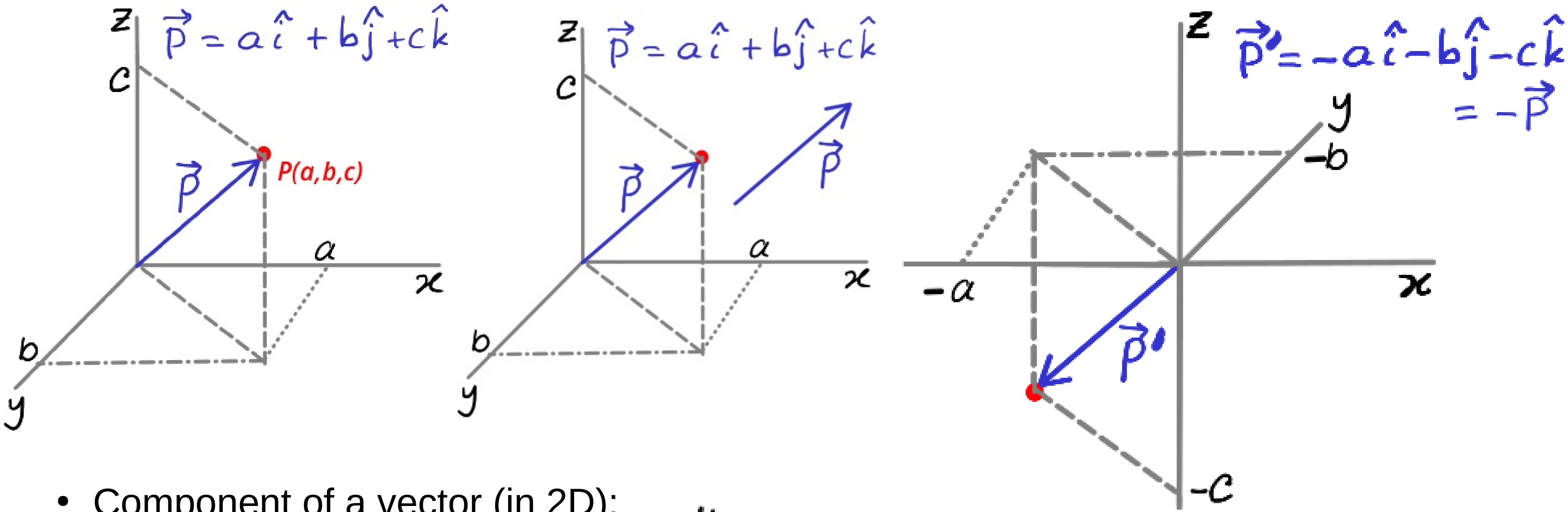
- Unit vector has megnitude (length) 1.
  Unit vector in the direction of vector ***A*** = ***A*/** |***A***|
  where |***A***| is the magnitude of vector ***A***





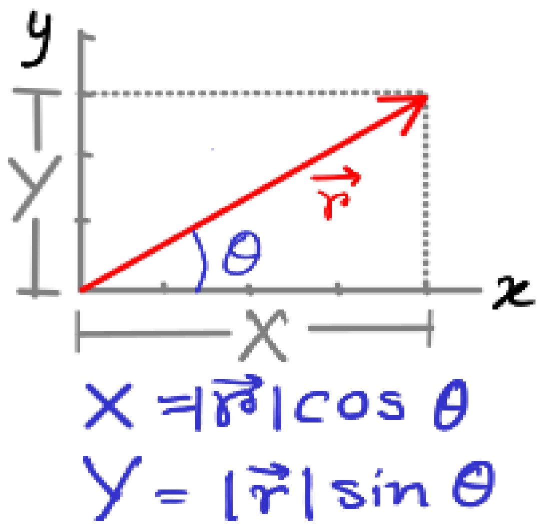


Cartesian Coordinate system



- Component of a vector (in 2D):

  Magnitude of vector ***r*** = |***r***|
  x- component: X = |***r***| *cos θ*
  y - component: Y = |***r***| *sin θ*



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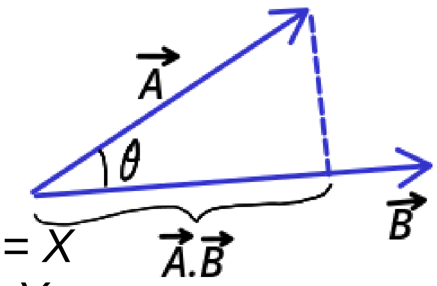
Products of vector

1) **Dot product** (scalar product):



Definition: ***A . B*** = |***A***||***B***| *cos θ*

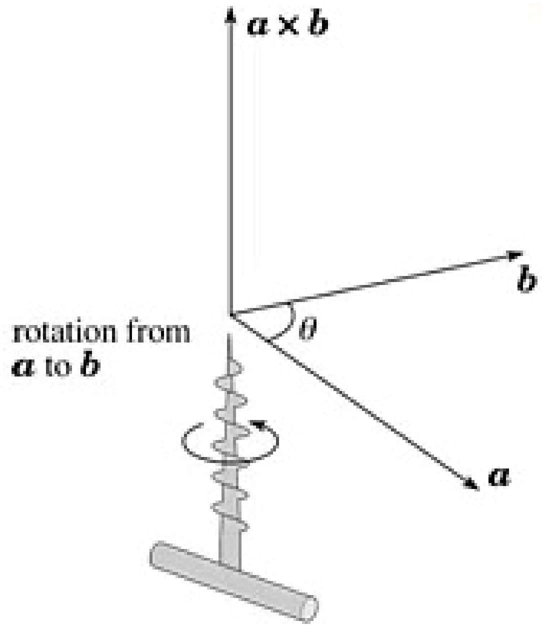
Dot product with unit vector in x- direction: ***r . i* = |*r*| |*1*|** *cos θ* =|***r***| *cos θ = X*
Dot product with unit vector in y- direction: ***r . j* = |*r*| |*1*|** *sin θ* =|***r***| *sin θ = Y*

*Dot product gives the component of **A** in the direction of **B.***

A . B = ***B . A*** *is a scalar quantity.*

$$\vec{A}\cdot\vec{A} = AA\cos 0 = A^2 = |\vec{A}|^2 \Rightarrow A = |\vec{A}| = \sqrt{\vec{A}\cdot\vec{A}}$$

2) **Cross product** (Vector product):



Definition: ***A*** *x* ***B*** = |***A***||***B***| *sin θ* ***n***

A *x* ***B* = -** ***B*** *x* ***A*** is a vector having direction ***n,*** which is perpendicular to the plance of ***A*** & ***B***.

$$\vec{A}\times\vec{A} = 0$$

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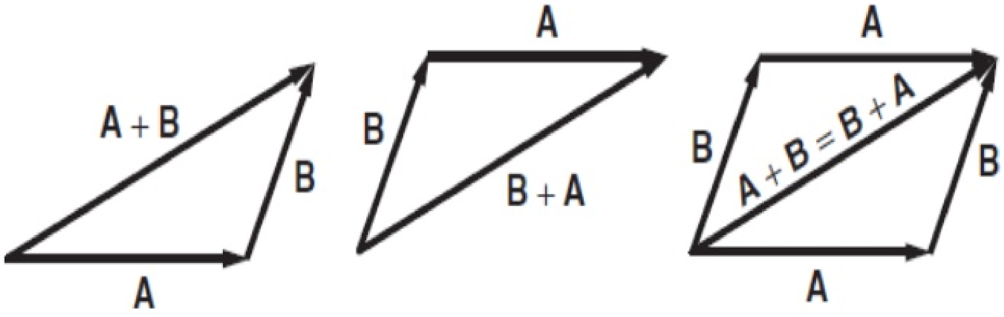
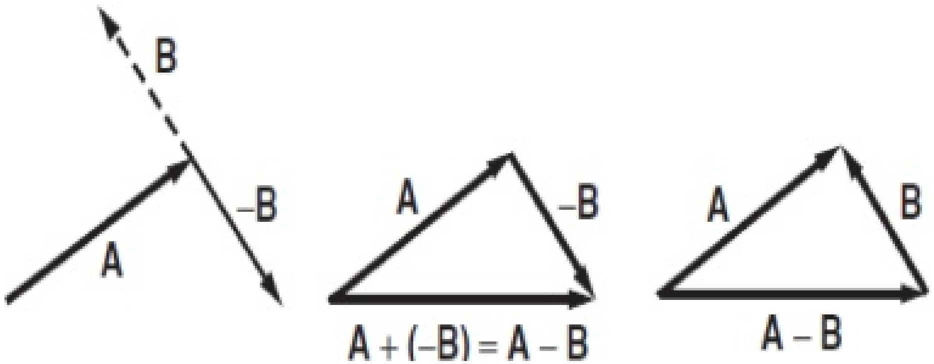
Vector addition

Addition:



Subtraction:

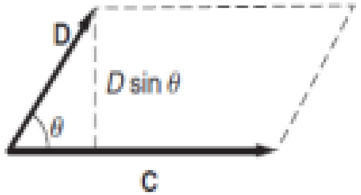


- Commmutative property of vector addition: ***P***+***Q* = *Q***+***P***
- Associative property: *(**P**+**Q**)+**R** = **P**+(**Q**+**R**),* *c(d**A**) = d(c**A**)=dc**A***

Application in Physics

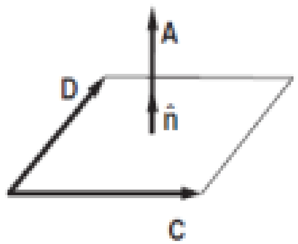
- Work done W, due to a force $\vec{F}$, causing displacement $\vec{d}$ is given by $W = \vec{F} \cdot \vec{d}$
- Torque $\vec{\tau}$, due to force $\vec{F}$ applied at a point whose position vector with respect to the reference point is $\vec{r}$ is $\vec{\tau} = \vec{r} \times \vec{F}$
- Force $\vec{F}$ acting on a charged particle with charge q, moving with velocity $\vec{v}$, exposed to a magnetic field $\vec{B}$, is given by $\vec{F} = q(\vec{v} \times \vec{B})$
- Even the surface area can be defined as a vector, in terms of a cross product
- Consider a parallelogram as shown here, the area can be written as

$A = base \times height = CD \sin\theta = |\vec{C} \times \vec{D}|$



- The direction is chosen to be one of the outward drawn normal $\hat{n}$, so that

$\vec{A} = |\vec{C} \times \vec{D}|\hat{n}$



Cartesian Coordinate system - I

- The simplest coordinate system in 3D is the Cartesian system which is defined by three mutually perpendicular directions x , y, and z .
- A vector $\vec{A}$, in this system, is defined by three components A_x , A_y , and A_z .
- Symbolically we can represent the vector $\vec{A}$, in terms of its components, as $\vec{A} = (A_x , A_y , A_z)$
- We can also define a set of unit vectors $\hat{\imath}$, $\hat{\jmath}$, and $\hat{k}$, in x , y , and z directions, respectively.
- Unit vectors $\hat{\imath}$, $\hat{\jmath}$ and $\hat{k}$, are called basis vectors of the 3D Cartesian coordinate system
- Now we can express $\vec{A}$, as a linear combination of these basis vectors

$$\vec{A} = A_x \hat{\imath} + A_y \hat{\jmath} + A_z \hat{k}$$

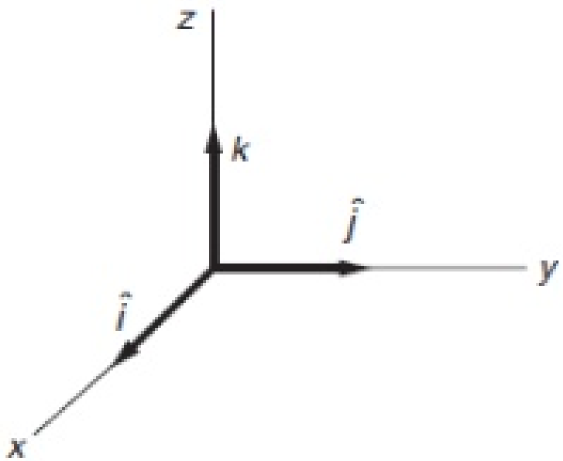
- Because these basis vectors are perpendicular to each other, they satisfy

$$\hat{\imath} \cdot \hat{\imath} = \hat{\jmath} \cdot \hat{\jmath} = \hat{k} \cdot \hat{k} = 1$$

$$\hat{\imath} \cdot \hat{\jmath} = \hat{\jmath} \cdot \hat{k} = \hat{k} \cdot \hat{\imath} = 0$$

$$\hat{\imath} \times \hat{\imath} = \hat{\jmath} \times \hat{\jmath} = \hat{k} \times \hat{k} = 0$$

$$\hat{\imath} \times \hat{\jmath} = \hat{k}; \hat{\jmath} \times \hat{k} = \hat{\imath}; \hat{k} \times \hat{\imath} = \hat{\jmath}$$



- Using this, it is easy to verify the following

$$\vec{A} + \vec{B} = (A_x + B_x)\hat{\imath} + (A_y + B_y)\hat{\jmath} + (A_z + B_z)\hat{k} = \vec{B} + \vec{A}$$

$$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z$$

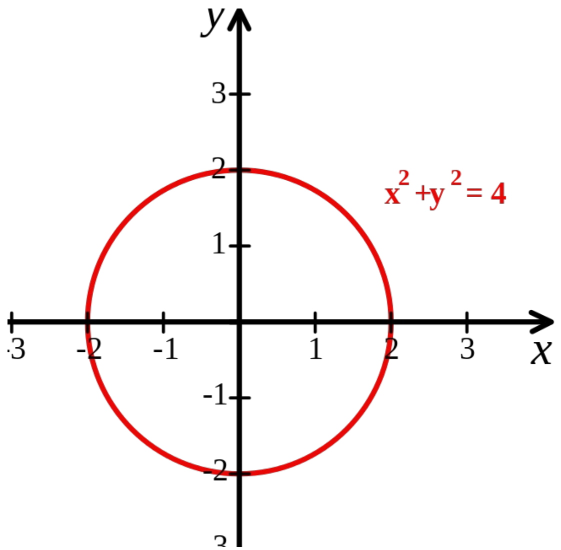
$$|\vec{A}| = \sqrt{\vec{A} \cdot \vec{A}} = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

Coordinate systems

- There are different types of coordinate systems based on the convenience to describe the coordinates with certain symmetries.

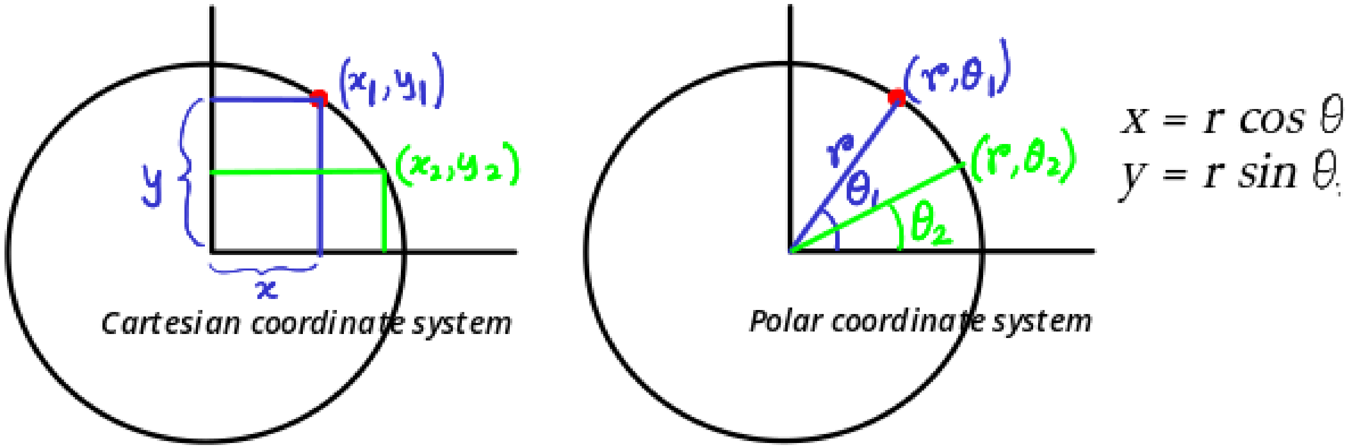
- Cartesian coordinate system: described by 3 mutually perpendicular axes. A point in Cartesian coordinate system is described by set of 3 numbers (x,y,z)

- Polar coordinate system: When there is a rotational symmetry present in configuration or motion, it is easier to describe through polar coordinate system.



Polar coordinate system - I

- Polar coordinate system in a plane, that is in 2D, is called **plane polar coordinate system**.
- In polar coordinate system, a point in 2D is specified by the coordinates *r, θ* where *r* is the distance of the point from origin, and *θ* is the angle in the line connecting the point and the origin makes with *x* axis.



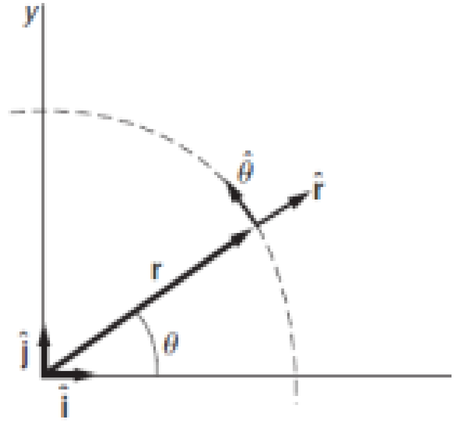
- From the above two figures, it is evindent that x = r cos *θ* and y = r sin *θ* and

$$r = \sqrt{x^2 + y^2} \text{ and } \theta = \tan^{-1} \frac{y}{x}$$

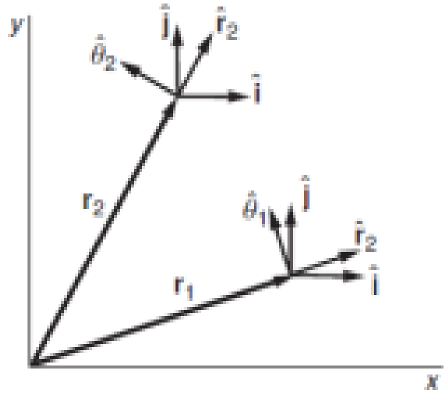
Polar coordinate system - II

- The unit vectors $\hat{r}$ and $\hat{\theta}$ can be described in the following way.
- Direction of $\hat{r}$ is the one in which r increases, but θ is held fixed.
- Similarly, $\hat{\theta}$ is in the direction in which θ increases, but r is held fixed
- Yet $\hat{r}$ and $\hat{\theta}$ are mutually perpendicular, just like $\hat{\imath}$ and $\hat{\jmath}$.
- Also note that unlike Cartesian coordinates, (r, θ) have different dimensions.

r has dimensions of length, while θ is dimensionless.



- In Cartesian coordinates, directions of unit vectors $\hat{\imath}$, $\hat{\jmath}$ and $\hat{k}$ are fixed in space, and same everywhere. However, this is not true in plane polar coordinates.



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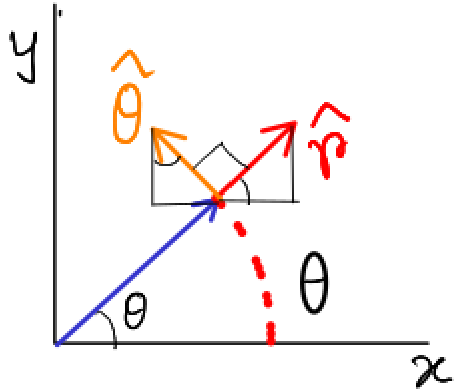
Cartesian and plain polar coordinate system

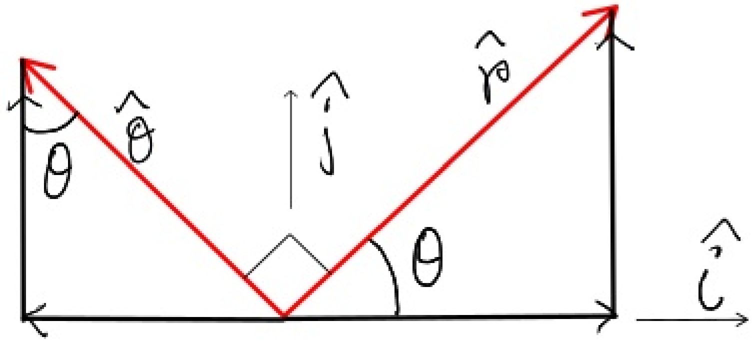
How to convert vectors from Cartesian to plain polar coordinates ?

- Position vector of an arbitrary point in two coordinate system is given by

$$\vec{r} = x\hat{i} + y\hat{j} = r\hat{r}$$

- Infinitesimal displacement is given by $d\hat{r} \;=\; dx\,\hat{i} + dy\,\hat{j} \;=\; dr\,\hat{r}$





$$\hat{r} = \cos\theta\,\hat{i} + \sin\theta\,\hat{j}$$
$$\hat{\theta} = -\sin\theta\,\hat{i} + \cos\theta\,\hat{j}$$

$$\hat{i} = \cos\theta\,\hat{r} - \sin\theta\,\hat{\theta}$$
$$\hat{j} = \sin\theta\,\hat{r} + \cos\theta\,\hat{\theta}$$